Around the Cusp singularity and the Breaking of Water Waves

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Abstract

Even if we extensively use them to communicate, light waves and sound waves stay more or less invisible to us. We can hear or see a signal or an image but during their transmission or their propagation through air, water or even vacuum for light, waves stay completely invisible to us. This is so true that in a physics class on waves, water waves are commonly used to illustrate the basic laws of wave propagation, reflexion or diffraction. Therefore, the idea of producing caustics and cusps when focusing hydrodynamical waves seems natural. Amazingly, it is only quite recently that this analogy between light waves and hydrodynamic waves was pointed out in the general context of pattern formation [Y. Pomeau, Europhys. Lett. 11, No 8 (1990)]. Later, She et al. [K. She, C.A. Greated and W.J. Easson, Applied Ocean Research. 19 (1997)] were the firsts to explicitly prepare a wave field where the wave energy is focused by an appropriate choice of the phase lag between the 75 paddles they used to generate the waves. More recently, thanks to the increased power of super computers, the full three dimensional numerical simulation of wave focusing has been carried out in order to simulate freak wave generation [C. Bradini and S. Grilli, Proc. 11th Offshore and polar engng. Conf. (ISOPE 2001), Stavanger, Norway, Vol 3, (2001)]. Inspired by these studies, we present here the very first moments of the breaking of surface waves. Using a parabolically shaped wave maker, we focus shallow water waves in a region of the water surface called the Huygens Cusp in optics. At this cusp,

the amplitude of the waves is increased by focusing and this leads to their breaking which is a typical property of water surface waves. We record these breakings using a fast video camera at a rate of 2000 images per second. The movie shows the very early time of the water tongue plunging ahead of the wave crest. Soon after, some capillarity wavelets are clearly visible. The image analysis of these space time data permits the measurement of the expected 3/2 power of time law as dictated by the cusp singular geometry given by the Catastrophe Theory [R. Thom, Stabilité structurelle et morphogénèse (Benjamin, Reading (Mass), 1972)]. To our knowledge this is the first time that this scaling law is measured from fluid dynamics videos.